PURSUING MATHEMATICS IN INDIA

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It is difficult for someone who is passionately involved in a pursuit to take a dispassionate view of the pursued; nevertheless I will try to give an objective view of the subject in this piece.

Mathematics may mean a motley different ideas to different people: to a poet it might be the choice of a metre, to a musician the harmony of the harmonics, to an accountant the balancing of sheets and to an anthropologist the rate of decay among the dead. Even though, often these ideas are not rigorous and not necessarily well-founded, every rational being still possesses some innate mathematical sense. For instance, the early human being must have had methods to decide whether his tribe had more strength than the opposing one. He must have come to realise that it is more cumbersome to measure it each and every time by direct comparison and it may be better to associate it with an abstract quantity which makes it more amenable for the purpose: a single word, a symbol or a sound for each measure of strength. In modern language we may think of them as the the natural numbers (1,2, 3 and so on). This abstract idea of natural numbers could now have been used to count the cattle, gauge the stock of their weaponry, or whether food and resources were being fairly distributed. Modern mathematics, of course, goes much beyond this idea which seems rather simple now. Personally I think it is a very sophisticated idea which possibly revolutionised the economy back then, setting the path to barter trade and such. The abstraction of taking a group of objects and replacing them by a number which no longer represents the group of objects but only its strength in many ways exemplifies the essence of mathematics and the power of abstraction.

However this describes only one side of the story: here we make a concrete idea more abstract so that it may become applicable elsewhere, giving us a bigger picture but still retaining some applications. The chief driver of mathematics is often not applications and usability, but a much deeper human tendency called curiosity. Having explored the world of integers and rationals; ancient civilisations in Greece, India and probably everywhere else stumbled upon the need for other numbers so as to measure, for instance, the length of the hypotenuse of a right-angled triangle of which the perpendicular sides have unit length. This would eventually pave the way to the realisation that if the perpendicular sides of the triangle have lengths a and b units, the length of the hypotenuse is $\sqrt{(a^2+b^2)}$. One may (and many have) now consider the question about whether, given that a and b are integers, the hypotenuse too can have integer length. Indeed they can; for instance if $a = 3$ and $b = 4$ then the length of the hypotenuse is 5. Thus now one may ask for a more natural way to generate all triples of integers (a, b, c) which can form the sides of a right angled triangle, or in other words, solve the equation: $a^2+b^2 = c^2$. A beautiful description is obtained of such triples and go by the name Pythagorean triples (https://en.wikipedia.org/wiki/Pythagorean_triple). A further meditation now might ask for solutions of the type $a^n + b^n = c^n$. This indeed was asked by a mathematician (and a lawyer) called Pierre de Fermat in 1637, only to be answered, after *backbreaking* labour of generations of mathematicians, by Andrew Wiles in 1995. This was after 358 years of development! The years spent in trying to answer this question may not always have succeeded in furthering our cause but did cultivate the fertile tracts of many mathematical fields. To say any further is beyond the interests and the scope of both this piece and my intellect.

This story, even as given, is both incomplete and insufficient in capturing the beauty of mathematical thought. There are many other beautiful fields, applications and theories which I have no

space for. I hope that you will pursue these in the future; for now I close this matter and turn to more practical issues:

How to prepare? Notice the emphasis on the word backbreaking in describing the 358 years in between 1637 and 1995. While the pursuit of mathematics can be joyous and entertaining, a rigorous and systemised study is a must for everyone; the romanticism of a lonely man in a room chipping away at his ideas fits well for movies but not for the real life.

Here are some books which might help you. They are not meant to improve your chances in the entrance exams but might give you a better feeling of what mathematics is about and introduce you to its culture. I am personally not much of a reader; these are mostly suggestions by some friends.

Some motivational/cultural books:

(1) How to Solve it by George Pólya (https://notendur.hi.is/hei2/teaching/Polya_HowToSolveIt.pdf)
(2) Mathematics can be Fun by Yakov Perelman (https://archive.org/details/MathematicsCanBeFun-1 and https://archive.org/download/AlgebraCanBeFun)
(3) What is Mathematics by Herbert Robbins and Richard Courant (https://yakovenko.files.wordpress.com/2009/11/cr.pdf)
(4) The Mathematical Century: The 30 Greatest Problems of the Last 100 Years by Piergiorgio Odifreddi (http://press.princeton.edu/titles/7789.html)
(5) Math Girls Series by Hiroshi Yuki (http://bentobooks.com/mathgirls/)

For problem solving one may consider the following books; they have many challenging questions which arent particularly run-of-the-mill. Do not be disheartened if you find these books too hard; look at them as distractions, puzzles or amusements.

(1) Problem Solving Strategies by Arthur Engel (http://www.springer.com/gp/book/9780387982199)
(2) Challenges and Thrills of Pre College Mathematics by C.R. Pranesachar, B.J. Venkatachala, K.N. Ranganathan and V. Krishnamurthy (this is not just a problem book but also a good book for gaining depth in the subjects discussed)
(3) Problems in Plane Geometry by I.F.Sharygin (https://www.slideshare.net/tapioca57/problems-inplanegeometrysharygin)

For the pure purpose of the entrance examinations at the Indian Statistical Institute one may have a look at the book "Test of Mathematics at the 10+2 level".

You should certainly consider:

(1) Math Olympiads: http://www.nbhm.dae.gov.in/olympiad.html
(2) KVPY: http://kvpy.iisc.ernet.in/main/selection.htm
(3) Bhaskaracharya Pratishthana: https://www.bprim.org/ (if you are close to Pune)

0.1. **Where to Go?** Here I mention a few which I am familiar with.

(1) Indian Statistical Institute and Chennai Mathematical Institute (http://www.isibang.ac.in/~statmath/courses/bmath.html and https://www.cmi.ac.in/admissions/) :Challenging examinations but wonderful places for mathematics.
(2) IISc Bangalore (http://www.iisc.ernet.in/ug/): This is a very good option if you want to explore some of the sciences as well.
(3) IIT Kanpur and Bombay (https://en.wikipedia.org/wiki/Joint_Entrance_Examination) IISERs (For instance in Pune and Mohali https://www.iiseradmission.in/)
(4) Most of the state universities (e.g., Delhi, Bombay, Calcutta, Tezpur, Hyderabad) do give an undergraduate mathematics degree.

0.2. **What can be done with a mathematics degree?** In the 18th century there were mathematicians who died in penury and hunger. It is no longer so. Mathematics has long been recognised as a useful tool in the sciences and the financial world. Most financial and IT firms, companies related to technological development and logistics management require people with well-developed mathematical skills. This being said, I will dissuade a student who is sure of corporate dreams from pursuing mathematics. It is not an easy subject and the monetary payoff is not necessarily comparable to other fields. Curiosity needs to be the central reason for the pursuance of mathematics; only with changing interests and opportunities later one may deviate from the initial goal of research and teaching.

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